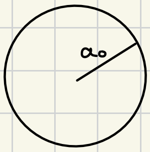
Apunts 7/02

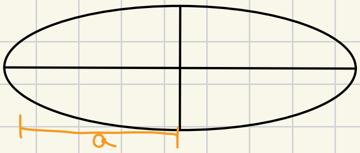
$E = ct$
$L = ct$

$\vec{l} = \vec{r} \times m\vec{v}$

$E = E_c + U(r)$



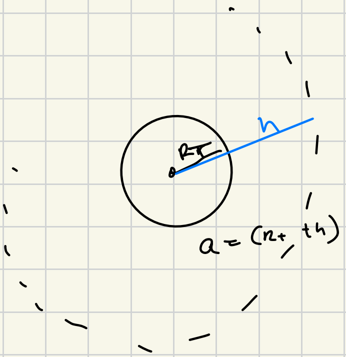
$E = \frac{K}{2r} = \frac{K}{2a_0}$



$E = \frac{K}{2a}$

Pot ser útil recordar això quan tinguis prob canvi òrbites:

$K = -gMm$ → K en cas de tenir massa m orbitant voltant de una M



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quan em donen U(r) puc utilitzar això: $E = \frac{1}{2}m\dot{r}^2 + \underbrace{\frac{L^2}{2mr^2} + U(r)}_{U_{ef}}$

de $U_{ef}' = 0$ → coincideix amb el radi de la òrbita circular associada

Exercici foto

$U(r) = Kr^2$ $\quad K > 0$

$L^2 = 2md^4K$ $\quad d, K > 0$

Quan em pregunten el camp, em pregunten la força:

$$\vec{F} = -\frac{dU(r)}{d\vec{r}}$$

perq (audio) per signe negatiu

$$F(r) = -\frac{dU(r)}{dr} = -K2r$$

atractiu

ⓑ Quan ens donguin $U(r)$ (enunciat) per trobar la E utilitzarem les coord polars:

$U(r) \rightarrow$ ens ho don enunciat!!!

$$E = r^2 + \frac{L^2}{2mr^2} + \overbrace{kr^2}^{U(r)}$$

$$U_{ef}(r) = \frac{L^2}{2mr^2} + kr^2$$

• Derivant i igualant $U_{ef}(r)$:

$$U_{ef}(r)' = \frac{L^2}{\cancel{2}m}(-\cancel{2})\, r^{-3} + 2kr = 0$$

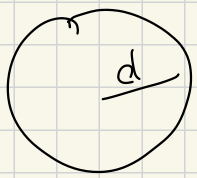
$$-\frac{L^2}{mr^3} = 2kr \quad ; \quad \frac{L^2}{m} = 2kr^4$$

$$\frac{\cancel{2}\cancel{m}\, d^4 \cancel{k}}{\cancel{m}} = \cancel{2}\cancel{k}\, r^4 \Longrightarrow d^4 = r^4$$

$$\boxed{r = d}$$

no posaré $\frac{K}{2r} = \frac{K}{2d}$!!! ...

no dic at wickyen



$$E = \frac{\cancel{2m}\, d^{\cancel{4}\,2} k}{\cancel{2m}\, d^{\cancel{2}}} + kd^2 = d^2k + kd^2 = 2kd^2$$

$\dot{r} = 0$ en orbita circular

$\dot{r} = 0$ en $\rightarrow$ apojeo, $\rightarrow$ perijeo

ⓒ calcul període:

$$T = \frac{2\pi}{\omega \left[\left(\frac{v}{r}\right)\right.} = \frac{2\pi r}{v} = \frac{2\pi d}{v}$$

el que em de calcular

si entiques en el cas de gravitatori ??

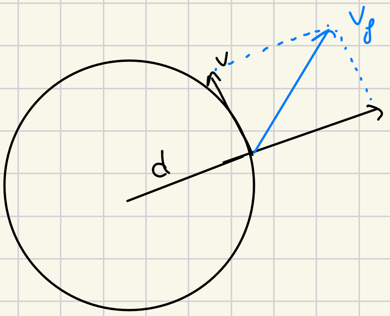
• calcul v

$$E = \frac{1}{2}mv^2 + U(r)$$

$$2kd^2 = \frac{1}{2}mv^2 + kd^2 \Longrightarrow kd^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2kd^2}{m}}$$

$$T = 2\pi d\sqrt{\frac{m}{2kd^2}} = 2\pi\sqrt{\frac{m}{2k}}$$

d)

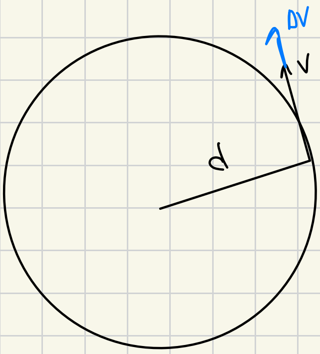


$\underbrace{\Delta v = 2d\sqrt{\frac{k}{m}}}$ velocitat radial (del radi)

Faig pitàgores: $v_f = \sqrt{v^2 + \Delta v^2} = \sqrt{\frac{2kd^2}{m} + \frac{4d^2k}{m}} = \sqrt{\frac{6kd^2}{m}}$

Calculo la E de la òrbita.

$$E_f = \frac{1}{2}mv^2 + U(r) = \frac{1}{2}\cancel{m}\cdot\frac{6kd^2}{\cancel{m}} + kd^2 = 3kd^2 + kd^2 = 4kd^2$$



$$v_f = v + \Delta v = \sqrt{\frac{2kd^2}{m}} + 2d\sqrt{\frac{k}{m}} =$$

$$= \sqrt{\frac{2kd^2}{m}} + \sqrt{\frac{4d^2k}{m}} = \sqrt{\frac{2kd^2}{m}}\,(1+\sqrt{2})$$

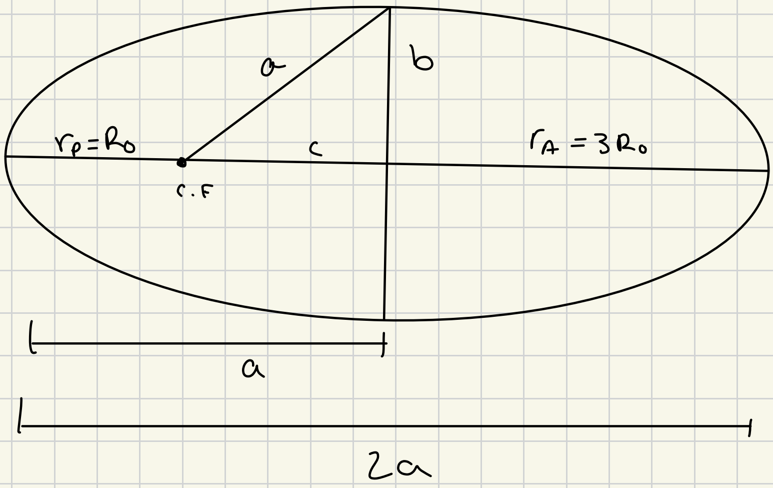
$$E_f = \frac{1}{2}m\cdot\frac{2kd^2}{m}(1+\sqrt{2})^2 + kd^2 = kd^2(1+\sqrt{2}) + kd^2$$

$$E_f = kd^2\left[(1+\sqrt{2})^2 + 1\right] = kd^2(1+2\sqrt{2}+2+1) = kd^2(4+2\sqrt{2})$$

Quina ha de ser la vel. de la partícula per a que l'at. s'ionitzi?

↳ haurà de per que la $\boxed{E=0}$ $\Longrightarrow 0 = E_c + E_p$

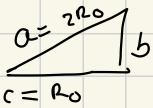
Exercici el·líptica



$r_P = R_0$
$r_A = 3R_0$
$2a = 3R_0 + R_0 = 4R_0$
$a = 2R_0$

ⓐ $\varepsilon = \dfrac{c}{a} = \dfrac{R_{\cancel{0}}}{2R_{\cancel{0}}} = 1/2$

ⓑ $a = 2R_0 \implies$ Eix major : $4R_0$

$b\,? \implies$ 

$\implies a^2 = b^2 + c^2$

$b^2 = a^2 - c^2$

$b^2 = 4R_0 - R_0^2 \implies b = R_0\sqrt{3}$

ⓒ sol en cas e^- àt H !!!

$E = \dfrac{K}{2a} = \dfrac{K}{2 \cdot 2R_0} = \dfrac{K}{4R_0} = -\dfrac{e^2}{4\pi\varepsilon_0 4R_0} = -\dfrac{e^2}{16\pi\varepsilon_0 R_0}$

ⓓ Sabent que en apogeo o perigeo $\dot{r} = 0$, llavors:

pq és e^- àt H

$$E = \frac{1}{2}\cancel{m\dot{r}^2}_{\to 0} + \frac{L^2}{2mr^2} + \frac{K}{r}$$

$$\frac{K}{2a} = \frac{L^2}{2mr^2} + \frac{K}{r} \implies \frac{K}{4R_0} = \frac{L^2}{2mR_0^2} + \frac{K}{R_0} \; ; \; \frac{K}{4R_0} - \frac{K}{R_0} = \frac{L^2}{2mR_0^2}$$

$$-\frac{3K}{4R_{\cancel{0}}} = \frac{L^2}{2mR_0^{\cancel{2}}} \implies L^2 = \frac{-3KmR_0}{2} \implies L = \sqrt{\frac{-3KmR_0}{2}} = \boxed{\sqrt{\frac{3e^2 m R_0}{8\pi\varepsilon_0}}}$$

don lo mateix en perigeu i apogeu

ⓔ $E = E_c + U(r)$

$$\frac{K}{2a} = \frac{1}{2}mv^2 + \frac{K}{r} \Rightarrow \frac{K}{4R_0} = \frac{1}{2}mv_p^2 + \frac{K}{R_0} \Rightarrow \frac{K}{4R_0} - \frac{K}{R_0} = \frac{1}{2}mv_p^2$$

$$\frac{K - 4K}{4R_0} = \frac{1}{2}mv_p^2 \Rightarrow v_p = \sqrt{\frac{-3K \cdot 2}{4R_0 m}} = \sqrt{\frac{-3K}{2R_0 m}} = \sqrt{\frac{3e^2}{8\pi\varepsilon_0 R_0 m}}$$

ⓔ

$$L_p = L_A \Rightarrow r_p \cancel{m} v_p \sin \cancel{90}_{1} = r_A \cancel{m} v_A \sin \cancel{90}_{1}$$

$$r_p V_p = r_A V_A \Rightarrow \cancel{R_0} V_p = V_A\, 3\cancel{R_0}$$

$$V_p = 3V_A \; ; \; V_A = \frac{V_p}{3} = \frac{1}{3}\sqrt{\frac{3e^2}{8\pi\varepsilon_0 R_0 m}}$$

entra a l'arrel elevant-se al quadrat !!!

$$V_A = \sqrt{\frac{e^2}{24\pi\varepsilon_0 R_0 m}}$$

ⓕ Velocitat mín va associada a una $E_g = 0$ ⚠

⚠ no és el mateix que sigui en apogeo q en perigeo.
→ on e^- es sent - atret és en perigeo (recordar ⊖K)

$$0 = \frac{1}{2}mV_{min}^2 + \frac{K}{R_0} \Rightarrow V_{min} = \sqrt{\frac{-K \cdot 2}{R_0 m}} = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 R_0 m}}$$

note.

Orbit ○

$E_c = ct$ pq la vel és la mateixa en tots els punts, ja q tots els punts es troben a la = dist

orbit ⬭

$E_m = ct$

$U(r) \rightarrow$ canvia pq la dist és ≠
K/⊖ → la dist va canviant en cada punt.

$E_c \rightarrow$ canvia pq la velocitat és ≠ en cada punt pq cada punt es troba a una dist ≠